Final Exam- Review 1 - Answers

Peyam Ryan Tabrizian

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1)

$$D = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \qquad P = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ -1 & 1 & 0 \end{bmatrix}$$

2)

$$\left\{ \frac{1}{\sqrt{2}} \begin{bmatrix} 0 \\ 1 \\ 1 \\ 0 \end{bmatrix}, \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ 0 \\ 0 \\ -1 \end{bmatrix}, \frac{1}{\sqrt{2}} \begin{bmatrix} 0 \\ -1 \\ 1 \\ 0 \end{bmatrix} \right\}$$

3) 0 (use the fact that $\int_{-\pi}^{\pi} \cos(mx)\sin(nx)dx = 0$ etc.)

4)

$$D = \begin{bmatrix} 0 & 0 \\ 0 & 10 \end{bmatrix}, \qquad P = \begin{bmatrix} \frac{3}{\sqrt{10}} & \frac{1}{\sqrt{10}} \\ \frac{1}{\sqrt{10}} & \frac{-3}{\sqrt{10}} \end{bmatrix}$$

5) $10y_2^2$, where $\mathbf{y} = P^T\mathbf{x}$, and P is the matrix in 4).

6)

$$A = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 3 & 1 \\ 0 & -1 & 3 \end{bmatrix}$$

7) $dim(V) = 3$, and:

$$\mathcal{B} = \left\{ \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \right\}$$

8) $Rank(A) = 2$

$$Col(A) = Span \left\{ \begin{bmatrix} 1 \\ -1 \\ 5 \end{bmatrix}, \begin{bmatrix} -4 \\ 2 \\ -6 \end{bmatrix} \right\}$$

$$Row(A) = Span \left\{ \begin{bmatrix} 1 \\ 0 \\ -1 \\ 5 \end{bmatrix}, \begin{bmatrix} 0 \\ -2 \\ 5 \\ -6 \end{bmatrix} \right\}$$

$$Nul(A) = Span \left\{ \begin{bmatrix} 2 \\ 5 \\ 2 \\ 0 \end{bmatrix}, \begin{bmatrix} -5 \\ -3 \\ 0 \\ 1 \end{bmatrix} \right\}$$

9) (a) **TRUE** (by rank-nullity theorem, $Rank(A) = n =$ number of columns)

(b) **FALSE** (again by rank-nullity theorem, the smallest possible dimension is 2, not 6)

(c) **TRUE** (again by rank-nullity theorem and using the fact that $dim(Nul(A)) = 0$)

(d) **TRUE** (use $Q^T Q = I$)

(e) **TRUE** (see the proof of theorem 1 on page 390)

(f) **TRUE** (Orthogonal projections)

(g) **TRUE** (If $A = PDP^{-1}$, then $A^2 = PD^2P^{-1}$, and notice D^2 is diagonal!)

(h) **FALSE** (For example, take $A = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$)

(i) **TRUE** (A is invertible, so $Row(A) = \mathbb{R}^n = Col(A)$)

(j) **TRUE** (at least I hope so :))